

# Abstract

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**Affiliation:** Kyushu University, Japan

**Title:** Unitary  $t$ -designs (a survey)

**Abstract:** The talk will be of very informal nature. (i) We start with an introductory survey on unitary  $t$ -designs, mainly following Roy-Scott [1]. We will consider only exact unitary  $t$ -designs from the viewpoint of mathematics. (ii) If a unitary  $t$ -design itself is a group, then it is called a unitary  $t$ -group. We review the complete classification of unitary  $t$ -groups in  $U(d)$  for all  $d \geq 2$  and  $t \geq 2$ , following Guralnick-Tiep [2] and Bannai-Navarro-Rizo-Tiep [3]. (iii) Then we review the recent preprint: Bannai-Nakahara-Zhao-Zhu [4] on the explicit constructions of unitary  $(t + 1)$ -designs from certain unitary  $t$ -groups. In particular, new exact unitary 4-designs in  $U(4)$  are explicitly constructed numerically. The full technical details of this will be presented in the talk of Da Zhao in this workshop. I hope my talk can serve as a gentle introduction to Da Zhao's talk. (iv) If time permits, I will discuss an ongoing joint attempt (with Da Zhao and Manabu Oura) to try to attack some problems mentioned in Zhu-Kueng-Grassl-Gross [5].

## References

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**Title:** Great antipodal sets on unitary groups and Hamming graphs

**Abstract:** The unitary group  $U(n)$  is a symmetric space and has the point-symmetry for every point  $g \in U(n)$ . A *great antipodal set* on  $U(n)$  is a “good” finite subset of  $U(n)$  related to the point-symmetries. We can obtain the Hamming graph  $H(n, 2)$  from a great antipodal set in a “natural way”. In this talk, we present some relations between harmonic analysis on  $U(n)$  and harmonic analysis on  $H(n, 2)$  in terms of design theory.

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**Title:** New lower bound on the sizes of designs

**Abstract:** Designs are good finite subsets that approximate a given space. On various spaces, we have a Fisher type lower bound on the sizes of designs. In this talk, I will discuss a new lower bound on the sizes of designs in a very general setup covering both combinatorial designs and geometric designs. On Johnson association schemes and real unit spheres, the new bound matches the Fisher type lower bound. On some union or product of association schemes, the new bound is better than the Fisher type lower bound, hence improves the known result. On unitary groups and projective unitary groups, the new bound is worse than the Fisher type lower bound, but we can use the fact that two bounds are different to prove the nonexistence of certain tight designs.

**Name:** Da Zhao, jasonzd@sjtu.edu.cn

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**Title:** On the explicit constructions of certain unitary  $t$ -designs

**Abstract:** Unitary  $t$ -designs are “good” finite subsets of the unitary group  $U(d)$  that approximate the whole unitary group  $U(d)$  well. If a unitary  $t$ -design itself is a group then it is called a unitary  $t$ -group. Although it is known that unitary  $t$ -designs in  $U(d)$  exist for any  $t$  and  $d$ , the unitary  $t$ -groups do not exist for  $t \geq 4$  if  $d \geq 3$ , as it is shown by Guralnick-Tiep (2005) and Bannai-Navarro-Rizo-Tiep (2018, preprint). Also, the explicit constructions of exact unitary  $t$ -designs in  $U(d)$  are not easy in general. In particular, it seems that the explicit constructions of unitary 4-designs in  $U(4)$  have been an open problem in physics (quantum information theory). We prove that some exact unitary  $(t + 1)$ -designs in the unitary group  $U(d)$  are constructed from unitary  $t$ -groups in  $U(d)$  that satisfy certain specific conditions. Based on this result, we specifically construct numerically exact unitary 3-designs in  $U(3)$  from the unitary 2-group  $SL(3, 2)$  in  $U(3)$ , and also unitary 4-designs in  $U(4)$  from the unitary 3-group  $Sp(4, 3)$  in  $U(4)$ . We also discuss some related problems.

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**Title:** Real equiangular line systems in low dimensions

**Abstract:** A system of lines through the origin of  $\mathbb{R}^d$  for which the angle between any pair of lines is a constant is called *equiangular*. A Seidel matrix, which can be interpreted as a variation of the adjacency matrix of a graph, is a tool for studying equiangular line systems. In this talk we present our recent improvements to the upper bounds for the cardinalities of equiangular line systems in low dimensions. A crucial ingredient for this improvement is a new restriction on the characteristic polynomial of a Seidel matrix.

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**Title:** Semidefinite programming bounds for spherical three-distance sets

**Abstract:** A spherical three-distance set is a finite collection  $C$  of unit vectors in  $\mathbb{R}^n$  such that for each pair of distinct vectors has three inner product values. This paper is devoted to use semidefinite programming method to improve the upper bounds of spherical three-distance sets for several dimensions. We improve the upper bounds for spherical three-distance sets in  $\mathbb{R}^7$  from 91 to 84 and we prove that maximum size of spherical three-distance sets is 2300 in  $\mathbb{R}^{23}$ .

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**Title:** On the multiplicities of digraph eigenvalues

**Abstract:** In 1977, Delsarte, Goethals, and Seidel showed that a regular (simple) graph on  $n$  vertices, whose  $(0, 1)$ -adjacency matrix  $A$  has the smallest eigenvalue  $< -1$  of multiplicity  $n - d$ , satisfies  $n \leq \frac{1}{2}d(d + 1) - 1$ . The bound is sharp, and it is known as the absolute bound if a graph is strongly regular.

In 2003, Bell and Rowlinson extended this bound to any eigenvalue of  $A$  distinct from 0 or  $-1$ , and showed that the graphs attaining equality are extremal strongly regular graphs (the only examples known are a pentagon, a complete multipartite graph, the Schläfli graph, the McLaughlin graph and their complements).

In this talk I will present the multiplicity bounds for eigenvalues of Hermitian adjacency matrices of digraphs.

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**Title:** Relative  $t$ -designs in Johnson association schemes for P-polynomial structure

**Abstract:** Relative  $t$ -designs are defined in both P- and Q-polynomial association schemes. In this talk, we will discuss our recent work on relative  $t$ -designs in Johnson association schemes  $J(v, k)$  for P-polynomial structure. It is known that each nontrivial shell  $X_r$  of  $J(v, k)$  is identified with the product of two smaller Johnson association schemes. We prove that relative  $t$ -designs in  $J(v, k)$  supported by one shell  $X_r$  are equivalent to weighted  $\mathcal{T}$ -designs in  $X_r$  for  $\mathcal{T} = \{(t_1, t_2) \mid 0 \leq t_1, t_2 \leq t\}$ . We also consider the existence of tight relative  $t$ -designs for  $t = 2, 3$ . We make an algorithm to construct such designs for  $(t, v, k, r) = (2, 14, 7, 3)$  and obtain examples for  $(t, v, k, r) = (3, 8u, 4u, 2u)$  with  $u \geq 2$ .

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**Title:** New constructions of biangular lines

**Abstract:** In this talk I will explore spherical few-distance sets in Euclidean space, that is, sets of real unit vectors whose pairwise inner products takes only “few” distinct values  $\{x, y, z, \dots\}$ . My focus will be mostly on the biangular case, that is, when this inner product set is of the form  $\{\pm x, \pm y\}$ . I will give various old and new constructions for biangular lines, and determine the largest sets in dimension six.

I will also explain how we discovered earlier the largest 4-distance sets in dimension three, the largest 3-distance sets in dimension four, and the largest 6-distance sets on the plane (both on the sphere, and in general position). A notable new result is that the platonic dodecahedron is the unique maximum spherical 5-distance set in dimension three.

The main tools are sophisticated computer algorithms for isomorph-free exhaustive generation of graphs, Gröbner basis computation for solving polynomial system of equations, and a compute cluster having 500 cores and 256GB of memory to perform all of these tasks in an efficient and reliable manner.

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**Title:** Efficient verification of bipartite and multipartite quantum states

**Abstract:** Bipartite and multipartite entangled states are of central interest to quantum information processing and foundational studies. Efficient verification of these states is a key to various applications, such as blind measurement-based quantum computation and quantum networks. Here we present optimal protocols or efficient protocols for verifying a number of important quantum states, including bipartite pure states, GHZ states, hypergraph states, and Dicke states. The connections with 2-designs and graph theory will be highlighted.

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**Title:** Unimodality and transformation semigroups

**Abstract:** Let  $S$  be a transformation semigroup acting on a set  $\Omega$ . The action of  $S$  on  $\Omega$  can be naturally extended to be an action on all subsets of  $\Omega$ . We say that  $S$  is  $\ell$ -homogeneous provided it can send  $A$  to  $B$  for any two (not necessarily distinct)  $\ell$ -subsets  $A$  and  $B$  of  $\Omega$ . On the condition that  $k \leq \ell < k + \ell \leq |\Omega|$ , we show that every  $\ell$ -homogeneous transformation semigroup acting on  $\Omega$  must be  $k$ -homogeneous. We report other variants of this result and suggest a matroid frame work for further research along the same direction.

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**Title:** Equiangular lines and the Lemmens-Seidel Conjecture

**Abstract:**

A system of lines through the origin in  $r$ -dimensional Euclidean space  $\mathbb{R}^r$  is called equiangular if the angle between any pair of lines is the same. Let  $N_\alpha(r)$  be the maximum number of a system of equiangular lines in  $\mathbb{R}^r$  with common angle  $\arccos \alpha$ . In 1973, Lemmens and Seidel conjectured  $N_{\frac{1}{5}}(r) = 276$  for  $23 \leq r \leq 185$  and  $N_{\frac{1}{5}}(r) = \lfloor \frac{r-5}{2} \rfloor + r + 1$  for  $r \geq 185$ . Let  $G$  be a graph on  $n$  vertices such that the corresponding Seidel matrix  $S(G)$  has smallest eigenvalue at least  $-5$ . We showed that, for  $n \geq 400$ , the rank of  $S(G) + 5\mathbf{I}$  is at least  $\lfloor \frac{2n+3}{3} \rfloor$ . This shows that the Lemmens-Seidel conjecture is true for  $n \geq 400$ . In this talk, we will discuss the proof. This is work in progress.

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**Title:** A future quantum internet: quantum data, security and machine learning

**Abstract:** The success of the modern internet relies in no small part on understanding the interplay between computation and security. More recently this area has also seen contributions from machine learning, including spam filters and malware detection. With the rising prevalence of machine learning algorithms, it is also important to address whether new security issues arise. Machine learning applications often require training or test data that originate from remote data centres or sensors. This decentralised set-up opens the door to adversaries that could exploit existing vulnerabilities in those algorithms. Evidence suggests that these vulnerabilities can grow with the dimensionality of the data. As quantum technologies become more accessible, these same concerns for quantum data is expected to become more important in a future quantum internet, especially as the most classically-intractable quantum systems of interest are usually high-dimensional. In this area which we can call adversarial quantum learning, we ask some basic questions: How do we verify high-dimensional quantum data with less resources? How do errors or adversarial changes in quantum data affect the probability of misclassification? Is there a dependence on dimensionality? How do we detect unusual quantum data? We take some first steps in addressing these questions and look towards the future of this intersection between computation and security on quantum data.

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**Title:** Control and simulation of large quantum systems

**Abstract:** In order to build a scalable quantum computer, we need to perform quantum error correction for constructing sufficiently clean quantum bits (qubits). To this end, we need several physical qubits with error rates smaller than a value called threshold. Constructing qubits with superconducting (SC) circuits is one of the most promising approaches to achieve such a large quantum system with a small error rate. Therefore, massive efforts have been paid for optimally controlling SC qubits. I will explain about two theoretical topics for developing SC qubits with small error rates.

The first topic is a method for optimizing control pulses of SC qubits. Quantum operations of SC qubits are typically performed by applying microwave control pulses to SC circuits. One of the most straightforward methods to optimize control pulses is monitoring the error rate of current control pulses and gradually modifying the shape of them to minimize the error rate. However, since simple methods require knowledge of exact physical models, it is practically hard to use them for optimizing controls of SC qubits. Here, we proposed a novel method to optimize control pulses, which we named as variational quantum gate optimization (VQGO). The key idea of VQGO is to use quantum circuits as an ansatz for control optimization, which enables practical optimization of control pulses for SC qubits. I will explain a construction of VQGO, and show that this method can optimize quantum operations which cannot be optimized with existing methods.

The other topic is an optimized classical simulator of quantum circuits. In order to evaluate the performance of control optimization methods, we need to test them under several noise models. However, it is typically hard to handle noise models in actual devices. Thus, we need to simulate noisy quantum circuits and test them with classical computer. Though simulation of quantum circuits requires costs which grow exponentially to the number of qubits, we can relax them by optimizing quantum circuits for classical simulation. I will explain how we can evaluate the time for simulating given quantum circuits, and propose a circuit optimization method which decreases time for simulating general quantum circuits.

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**Title:** Characterizing elemental operations for large-scale quantum computer

**Abstract:** Scientific research and development toward realizing a large-scale quantum computer have progressed significantly over the last twenty years, but much larger systems and higher accuracies are required for the realization. A reliable method for characterizing elementary quantum operations that is suitable for improving and validating their accuracies is indispensable for achieving higher accuracies. In this talk, first we review recent progress of development of quantum computer, especially based on superconducting quantum circuit systems. Second, we explain conditions that such a reliable method are required or expected to satisfy, and we also explain current standard methods, e.g., randomized benchmarking, quantum tomography, and gate-set tomography, are not sufficient from the viewpoint of the conditions. Third, we propose a new characterization method, called regularized self-consistent quantum tomography (RSCQT). We describe that it has superior properties such as (i) the high reliability guaranteed by the asymptotic convergence and convergence rate, (ii) applicability of the estimates to conventional validation protocols, and (iii) availability of detailed information of errors on the operations. Finally, we explain disadvantages of RSCQT, which have to be overcome in future.

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